Additional materials for the Sabina's stats series lecture "Who counts? Denominator, bias and illusion of equity"

Lecture recording (April 26, 2026): https://www.youtube.com/watch?v=zjakvEWA6WY

Summary:

During the lecture we talked about Pareto dominance theory.

This document includes brief description of key elements used in the lecture to help you understand it better.

Pareto Dominance

Pareto Dominance comes from economics and decision theory and is used to compare different policies or decisions. A policy is considered "Pareto Dominated" if there is another option that improves outcomes for at least one group without making anyone else worse off. In simple terms, it means there was a clearly better alternative available.

We can call it the "No Losers" Rule

An outcome Pareto dominates another if it meets two strict conditions:

- No one is worse off->In the new state, every individual is at least as well off as they were in the previous state
- At least one person is better off and has a strictly better outcome than before.

If you can improve someone's situation without hurting anyone else, you have made a Pareto Improvement. When you reach a point where it is impossible to help one person without hurting another, you have reached Pareto Optimality (or Efficiency).

This theory is the bridge between the "True Effect" and the concept of "Fairness" we talked about earlier

Efficiency (The Statistical Goal): Pareto dominance focuses strictly on whether an intervention (like a tutor) increases the total "value" (test scores) without decreasing it for anyone else.

The Fairness Conflict: A situation can be Pareto Optimal but highly unfair. For example, if a tutor only helps the top 1% of students and ignores the rest, that is a Pareto Improvement (the top 1% are better, no one else is worse). However, from an equity standpoint, this increases the gap between students.

Pareto Dominance in Causal Inference: When researchers look for the Average Treatment Effect (ATE), they are often checking if the treatment "Pareto dominates" the control. This means that on average, the intervention helps and doesn't cause harm

In the lecture, this concept was connected to fairness and equity research through the paper "Causal Conceptions of Fairness and their Consequences" by Hamed Nilforoshan, Johann Gaebler, Ravi Shroff, and Sharad Goel.

Link to the paper:
https://proceedings.mlr.press/v162/nilforoshan22a.html

The paper examines how strict mathematical definitions of fairness can sometimes produce worse real-world outcomes, even when they appear statistically "fair." The main argument is that fairness rules based only on mathematical symmetry may ignore important contextual information about structural disadvantage, access to resources, or baseline opportunities.

In the lecture, we discussed the university admission example. Two students have the same SAT score:

- Student A comes from a wealthy environment with tutors, strong schools, and educational resources.
- Student B comes from a disadvantaged environment with fewer opportunities and major structural barriers.

From a causal perspective, Student B may demonstrate greater underlying potential because the same outcome was achieved under much more difficult conditions. However, some strict fairness frameworks argue that admission decisions should remain unchanged even if characteristics linked to background or socioeconomic status are altered in the "counterfactual" world. To satisfy this mathematical definition of fairness, the system may be forced to ignore school quality, structural barriers, resilience, or access to opportunity.

This creates the Pareto problem: the system becomes mathematically "fair," but produces worse outcomes in practice.

The university may:

- admit a less suitable class overall,
- fail to recognize true potential,
- reward privilege instead of resilience,
- and reduce opportunities for the very groups the fairness rule was intended to protect.

This is why the lecture emphasized that fairness in research cannot rely only on mathematical formulas or statistical symmetry. Equity research requires understanding:

- confounding,
- denominator definition,
- structural bias,
- unequal access to opportunity,
- and the real-world consequences of analytical decisions.

This is why fairness in research cannot rely only on mathematical formulas or statistical symmetry. A model can satisfy a formal definition of fairness while still producing Pareto-dominated outcomes that fail to improve real-world conditions. The concept reminds researchers that equity research must balance statistical fairness with meaningful real-world impact, valid causal reasoning, and careful understanding of the underlying social structure.